$Re_L = \frac{u_\infty L}{\upsilon} = \frac{5*0.6}{15.96*10^{-6}} = 1.88 * 10^5$ therefore the flow is laminar

$$\overline{T_w - T_\infty} = \frac{q_w L/k}{0.6795 Re_L^{1/2} Pr^{1/3}}$$

$$\overline{T_w - T_\infty} = \frac{\left(\frac{1000}{0.6^2}\right)\left(0.6/0.02624\right)}{0.6795(1.88*10^5)^{1/2}(0.708)^{1/3}} = 241.85\ ^\circ\text{C}$$

$$T_w = 241.85 + 27 = 268.85\ ^\circ\text{C}$$

Now we find the properties at the film temperature

$$T_f = \frac{T_w + T_\infty}{2} = \frac{268.85+27}{2} = (147.927 + 273) = 421\ K$$

$\upsilon = 28.22 * 10^{-6}$ m^2/s, $Pr = 0.687$, $k = 0.035$ W/m.˚C.

$$Re_L = \frac{5*0.6}{28.22*10^{-6}} = 1.06 * 10^5$$

$$\overline{T_w - T_\infty} = \frac{\left(\frac{1000}{0.6^2}\right)\left(0.6/0.035\right)}{0.6795(1.06*10^5)^{1/2}(0.687)^{1/3}} = 243.6\ ^\circ\text{C}$$

$$Nu_x = \frac{q_w x}{k(T_w - T_\infty)}$$

$$T_w - T_\infty = \frac{q_w x}{k Nu_x}$$

$$T_w - T_\infty = \frac{q_w x}{k Nu_x}$$

$$Nu_x = 0.453\ Re_x^{1/2} Pr^{1/3}$$

$$T_w - T_\infty = \frac{q_w x}{(k) 0.453\ Re_x^{1/2} Pr^{1/3}}$$

At $x = L$

$$T_w - T_\infty = \frac{q_w L}{(k) 0.453\ Re_L^{1/2} Pr^{1/3}}$$

$$T_w - T_\infty = \frac{\left(\frac{1000}{0.6^2}\right)\left(0.6/0.035\right)}{0.453\ (1.06*10^5)^{1/2}(0.687)^{1/3}} = 365.9\ ^\circ\text{C}$$

EXAMPLE 5.4:Plate with Unheated Starting Length

Air at 1 atm and 300 K flows across a 20-cm-square plate at a free-stream velocity of 20 m/s. The last half of the plate is heated to a constant temperature of 350 K. Calculate the heat lost by the plate.

Solution

First we evaluate the air properties at the film temperature

$T_f = \frac{T_w+T_\infty}{2} = \frac{350+300}{2} = 325\ K$

$\upsilon = 18.23 * 10^{-6}$ m²/s, $k = 0.02814$ W/m.˚C, $Pr = 0.7$

At $x = L$

$Re_L = \frac{u_\infty L}{\upsilon} = \frac{20*0.2}{18.23*10^{-6}} = 2.194 * 10^5$

$Re_L < 5 * 10^5$

Therefore the flow is laminar

$$h_x = 0.332k\ Pr^{1/3}\left(\frac{u_\infty}{\upsilon x}\right)^{1/2}\left[1-\left(\frac{x_o}{x}\right)^{3/4}\right]^{-1/3}$$

$$h_L = 0.332 * 0.02814(\ 0.7)^{1/3}\left(\frac{20}{18.23*10^{-6}*0.2}\right)^{1/2}\left[1-\left(\frac{0.1}{0.2}\right)^{3/4}\right]^{-1/3}$$

$h_L = 26.253$ W/m².˚C.

$$\bar{h}_{x_o-L} = h_{x=L}\left(2L\frac{1-\left(x_o/L\right)^{3/4}}{L-x_o}\right)$$

$$\bar{h}_{x_o-L} = 26.253\left(2(0.2)\frac{1-\left(0.1/0.2\right)^{3/4}}{0.2-0.1}\right)$$

$\bar{h}_{x_o-L} = 42.566$ W/m².˚C

$q = \bar{h}_{x_o-L}(L-x_o) * w(T_w - T_\infty)$

$q = 42.566(0.2-0.1) * 0.2(350-300) = 42.566$ W

EXAMPLE 5.5: Oil Flow Over Heated Flat Plate

Engine oil at 20◦C is forced over a 20-cm-square plate at a velocity of 1.2 m/s. The plate is heated to a uniform temperature of 60◦C. Calculate the heat lost by the plate.

Solution

We first evaluate the film temperature:

$T_f = \frac{T_w+T_\infty}{2} = \frac{60+20}{2} = 40 + 273 = 313$ K

$\upsilon = 0.00024$ m²/s, $k = 0.144$ W/m.˚C, $Pr = 2870$, $\rho = 876$ kg/m³

$$Re_x = \frac{u_\infty x}{v} = \frac{1.2*0.2}{0.00024} = 1000$$

$$Nu_x = \frac{0.3387 Re_x^{1/2} Pr^{1/3}}{\left[1+\left(\frac{0.0468}{Pr}\right)^{2/3}\right]^{1/4}} = \frac{0.3387(1000)^{1/2}(2870)^{1/3}}{\left[1+\left(\frac{0.0468}{2870}\right)^{2/3}\right]^{1/4}} = 152.2$$

$$h_x = Nu_x \frac{k}{x} = 152.2 * \frac{0.144}{0.2} = 109.6 \text{ W/m}^2.\ ^\circ\text{C}$$

$$\bar{h} = 2h_x = 2 * 109.6 = 219.2 \text{ W/m}^2.\ ^\circ\text{C}$$

$$q = \bar{h}A(T_w - T_\infty) = 219.2\ *(0.2)^2(60-20) = 350.6 \text{ W}$$

5.5 The Relation Between Fluid Friction and Heat Transfer

We have already seen that the temperature and flow fields are related. Now we seek an expression whereby the frictional resistance may be directly related to heat transfer.

The shear stress at the wall may be expressed in terms of a friction coefficient C_f :

$$\tau_w = C_f \frac{\rho u_\infty^2}{2} \qquad 5.30$$

The exact solution of the boundary-layer equations yields

$$\frac{C_{fx}}{2} = 0.332 Re_x^{-1/2} \qquad 5.31$$

$$Nu_x = 0.332\ Pr^{1/3} Re_x^{1/2} \qquad 5.20\text{ a}$$

Equation (5.20 a) may be rewritten in the following form:

$$\frac{Nu_x}{Re_x Pr} = \frac{h_x}{\rho c_p u_\infty} = 0.332\ Pr^{-2/3} Re_x^{-1/2}$$

The group on the left is called the Stanton number,

$$St_x = \frac{h_x}{\rho c_p u_\infty}$$

So that

$$St_x Pr^{2/3} = 0.332 Re_x^{1/2} \qquad 5.32$$

Upon comparing Equations (5.31) and (5.32), we note that the right sides are alike except for a difference of about 3 percent in the constant, which is the result of the approximate nature of the integral boundary-layer analysis. We recognize this approximation

And write

$$St_x Pr^{2/3} = \frac{C_{fx}}{2} \qquad 0.6 < pr < 60 \qquad 5.33$$

Equation (5.33), called the *Reynolds-Colburn analogy,* expresses the relation between fluid friction and heat transfer for laminar flow on a flat plate. The heat-transfer coefficient thus could be determined by making measurements of the frictional drag on a plate under conditions in which no heat transfer is involved.

It turns out that Equation (5.33) can also be applied to turbulent flow over a flat plate and in a modified way to turbulent flow in a tube. It does not apply to laminar tube flow.

EXAMPLE 5.3: A 1.0-kW heater is constructed of a glass plate with an electrically conducting film that produces a constant heat flux. The plate is 60 cm by 60 cm and placed in an airstream at 27◦C, 1 atm with u_∞ =5 m/s. Calculate the average temperature difference along the plate.

EXAMPLE 5.6 Drag Force on a Flat Plate

For the flow system in Example 5.2 compute the drag force exerted on the first 40 cm of the plate using the analogy between fluid friction and heat transfer.

Solution

We use Equation (5.33) to compute the friction coefficient and then calculate the drag force. An average friction coefficient is desired, so

$$St_x Pr^{2/3} = \frac{C_{fx}}{2}$$

From example 5.2

$T_f = \frac{T_w + T_\infty}{2}$, $T_\infty = 27^\circ C$, $u_\infty = 2$ m/s, T_w=60˚C, $c_p = 1.006$ Kj/kg. ˚C.

$$T_f = \frac{60+27}{2} = 43.5 + 273 = 316.5 \text{ K}$$

$$\rho = \frac{p}{RT} = \frac{1.0132 * 10^5}{(287)(316.5)} = 1.115 \text{ kg/m}^3$$

For 40 cm length

From example 5.2

$\bar{h} = 8.698$

$$\bar{St} = \frac{\bar{h}}{\rho c_p u_\infty} = \frac{8.698}{1.115*1006*2} = 3.88 * 10^{-3}$$

$$\frac{\bar{C_f}}{2} = \bar{St} Pr^{2/3} = 3.88 * 10^{-3}(0.7)^{2/3} = 3.06 * 10^{-3}$$

$$\bar{\tau_w} = \bar{C_f}\frac{\rho u_\infty^2}{2} = 3.06 * 10^{-3} * 1.115 * (2)^2 = 0.0136 \text{ N/m}^2$$

$$D = \bar{\tau_w} L = 0.0136 * 0.4 = 5.44 \text{ N.m}$$

5.6 Turbulent-Boundary-Layer Heat Transfer

Schlichting has surveyed experimental measurements of friction coefficients for turbulent flow on flat plates. We present the results of that survey so that they may be employed in the calculation of turbulent heat transfer with the fluid-friction–heat-transfer analogy. The *local* skin-friction coefficient is given by

$$C_{fx} = 0.0592 Re_x^{-\frac{1}{5}} \qquad 5 * 10^5 < Re_x < 10^7 \qquad 5.34$$

$$C_{fx} = 0.370(\log Re_x)^{-2.584} \qquad 10^7 < Re_x < 10^9 \qquad 5.35$$

The *average-friction coefficient* for a flat plate with a laminar boundary layer up to Re_{crit} and turbulent thereafter can be calculated from

$$\bar{C_f} = \frac{0.455}{(\log Re_L)^{2.584}} - \frac{A}{Re_L} \qquad Re_L < 10^9 \quad \text{for laminar and turbulent} \qquad 5.36$$

where the constant A depends on Re_{crit} in accordance with Table 5.1. A somewhat simpler formula can be obtained for lower Reynolds numbers as

$$\bar{C_f} = \frac{0.074}{Re_L^{\frac{1}{5}}} - \frac{A}{Re_L} \qquad Re_L < 10^7 \qquad 5.37$$

Table 5-1

Re_{crit}	3×10^5	5×10^5	10^6	3×10^6
A	1055	1742	3340	8940

Applying the fluid-friction analogy

$$St\, Pr^{2/3} = \frac{Cf}{2}$$

we obtain the local turbulent heat transfer as:

$$St_x\, Pr^{2/3} = 0.0296\, Re_x^{-\frac{1}{5}} \qquad 5*10^5 < Re_x < 10^7 \qquad 5.38$$

Or

$$St_x\, Pr^{2/3} = 0.185(\log Re_x)^{-2.584} \qquad 10^7 < Re_x < 10^9 \qquad 5.39$$

The average heat transfer over the entire laminar-turbulent boundary layer is

$$\bar{St}\, Pr^{2/3} = \frac{Cf}{2} \qquad 5.40$$

For $Re_{crit} = 5 \times 10^5$ and $Re_L < 10^7$, Equation (5.37) can be used to obtain

$$\bar{St}\, Pr^{2/3} = 0.037 Re_L^{-1/5} - 871 Re_L^{-1} \qquad 5.41$$

$$\bar{St} = \frac{\overline{Nu}}{(Re_L \mathrm{Pr})}$$

$$\overline{Nu_L} = \frac{hL}{k} = Pr^{\frac{1}{3}}(0.037 Re_L^{0.8} - 871) \qquad 5.42$$

For higher Reynolds numbers the friction coefficient from Equation (5.36) may be used, so that

For $10^7 < Re_L < 10^9$ and $Re_{crit} = 5 \times 10^5$

$$Nu_L = \frac{\bar{h}L}{k} = [0.228 Re_L(\log Re_L)^{-2.584} - 871] Pr^{\frac{1}{3}} \qquad 5.43$$

If Re_{crit} differs from 5×10^5, An alternative equation is suggested by Whitaker that may give better results with some liquids because of the viscosity-ratio term:

$$\overline{Nu_L} = 0.036 Pr^{0.43}(Re_L^{0.8} - 9200)\left(\frac{\mu_\infty}{\mu_w}\right)^{\frac{1}{4}} \qquad 5.44$$

$0.7 < Pr < 380$

$2*10^5 < Re_L < 5.5*10^6$

$0.26 < \frac{\mu_\infty}{\mu_w} < 3.5$

μ_∞ : the viscosity evaluated at T_∞.

μ_w : the viscosity evaluated at T_w.

For the gases the viscosity ratio is dropped and the properties are evaluated at T_f.

Constant Heat Flux

For constant-wall-heat flux in turbulent flow that the local Nusselt number is only about 4 percent higher than for the isothermal surface; that is,

$$Nu_x = 1.04 Nu_x]_{T_{w=constant}} \quad 5.45$$

Example 5.7:Turbulent Heat Transfer from Isothermal Flat Plate

Air at 20◦C and 1 atm flows over a flat plate at 35 m/s. The plate is 75 cm long and is maintained at 60◦C. Assuming unit depth in the *z* direction, calculate the heat transfer from the plate.

Solution

We evaluate properties at the film temperature:

$$T_f = \frac{T_w + T_\infty}{2} = \frac{60+20}{2} = 40 + 273 = 313 \text{ K}$$

$$\rho = \frac{p}{RT} = \frac{1.0132 * 10^5}{(287)(313)} = 1.128 \text{ kg/m}^3.$$

$\mu = 1.906 * 10^{-5}$ kg/m.s, $k = 0.02723$ W/m.°C, $Pr = 0.7$, $C_p = 1.007$ kJ/kg.°C.

$$Re_L = \frac{u_\infty L}{v} = \frac{\rho u_\infty L}{\mu} = \frac{1.128 * 35 * 0.75}{1.906 * 10^{-5}} = 1.553 * 10^6 > 5 * 10^5$$

then the flow is turbulent

therefore we use equation 5.42

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = Pr^{\frac{1}{3}}(0.037 Re_L^{0.8} - 871)$$

$$\overline{Nu}_L = (0.7)^{\frac{1}{3}}[(0.037)(1.553 * 10^6)^{0.8} - 871] = 2180$$

$$\bar{h} = \overline{Nu}_L \frac{k}{L} = \frac{(2180)(0.02723)}{0.75} = 79.1 \text{ W/m}^2.\text{°C}.$$

$$q = \bar{h}A(T_w - T_\infty) = (79.1)(0.75)(60 - 20) = 2373 \text{ W}.$$

5.7 Turbulent-Boundary-Layer Thickness

The turbulent boundary layer thickness is calculated from the equation below:

1. The boundary layer is fully turbulent from the leading edge of the plate.

$$\delta = \frac{0.381x}{Re_x^{\frac{1}{5}}} \quad 5.46$$

2. The boundary layer follows a laminar growth pattern up to $Re_{critical} = 5 * 10^5$ and a turbulent growth thereafter.

$$\delta = \left(0.381 Re_x^{-\frac{1}{5}} - 10{,}256 Re_x^{-1}\right) x \qquad 5 * 10^5 < Re_x < 10^7 \qquad 5.47$$

Example 5.8:

Turbulent-Boundary-Layer Thickness

Calculate the turbulent-boundary-layer thickness at the end of the plate for Example 5.7, assuming that it develops (*a*) from the leading edge of the plate and (*b*) from the transition point at $Re_{crit} = 5 * 10^5$.

Solution

Since we have already calculated the Reynolds number as $Re_L = 1.553 \times 10^6$, it is a simple matter

to insert this value in Equations (5.46) and (5.47) along with $x = L = 0.75$ m to give

(a) $\delta = \frac{0.381x}{Re_x^{\frac{1}{5}}} = \frac{(0.381)(0.75)}{(1.553*10^6)^{\frac{1}{5}}} = 0.0165$ m= 16.5 mm

(b) $\delta = \left(0.381 Re_x^{-\frac{1}{5}} - 10{,}256 Re_x^{-1}\right) x$

$$\delta = \left[(0.381)(1.553 * 10^6)^{-\frac{1}{5}} - (10{,}256)(1.553 * 10^6)^{-1}\right] * 0.75 = 9.9 \text{ mm}$$

5.8 Internal Forced Convection

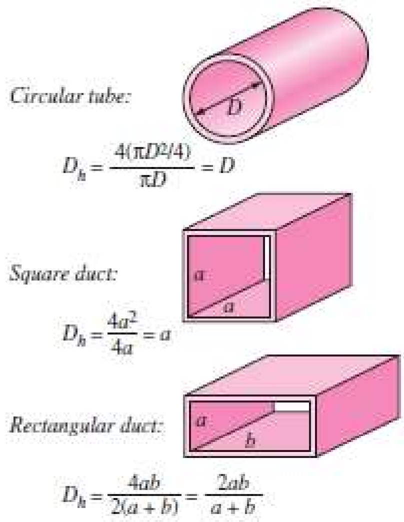
5.8.1 Heat Transfer in Laminar Tube Flow

Consider the flow in a tube as shown in Figure 5.7. A boundary layer develops at the entrance, as shown. Eventually the boundary layer fills the entire tube, and the flow is said to be fully developed. If the flow is laminar, a parabolic velocity profile is experienced, as shown in Figure 5.7*a*. When the flow is turbulent, a somewhat blunter profile is observed, as in Figure 5.7*b*. In a tube, the Reynolds number is again used as a criterion for laminar and turbulent flow. For

$$Re_{D_h} = \frac{u_m D_h}{v} > 2300 \qquad 5.48$$

Where

$$D_h = \frac{4A_c}{P}$$



the flow is usually observed to be turbulent D is the tube diameter.

Again, a range of Reynolds numbers for transition may be observed, depending on the pipe roughness and smoothness of the flow. The generally accepted range for transition is

$2000 < Re_D < 4000$

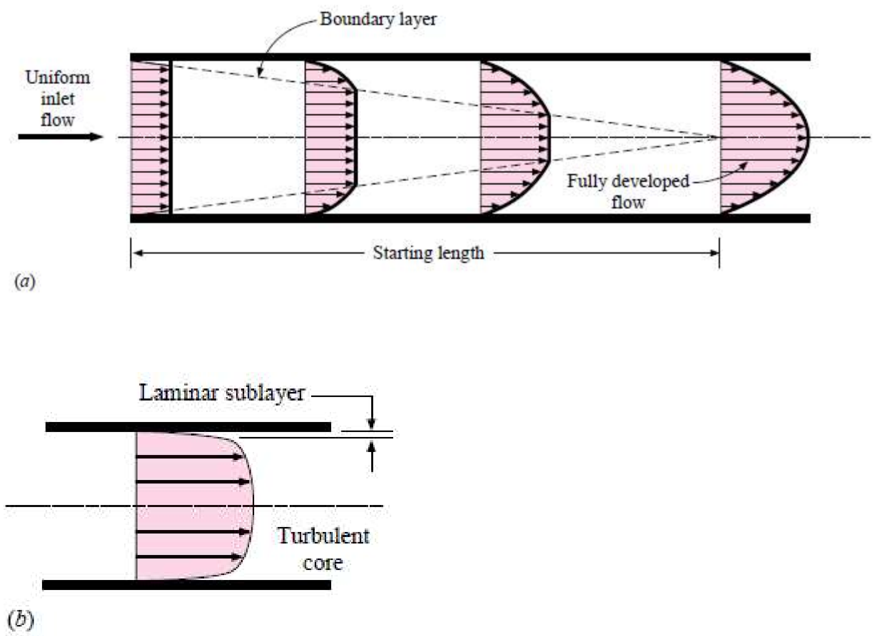


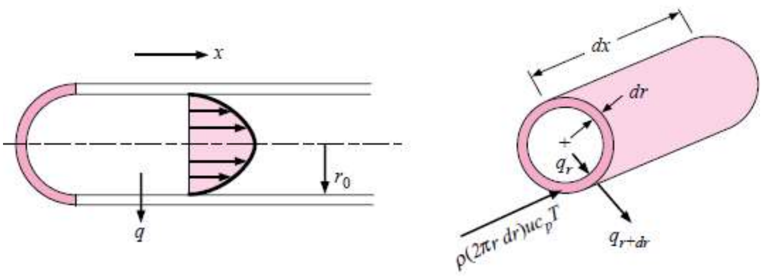
Figure 5.7 Velocity profile for (*a*) laminar flow in a tube and (*b*) turbulent tube flow.

Mean velocity

The value of the mean velocity u_m in a tube is determined from

$$\dot{m} = \rho u_m A_c = \int \rho\, u(r,x) dA_c \qquad 5.49$$

$$u_m = \frac{\int_0^{r_o} \rho u(r,x) 2\pi r dr}{\rho \pi r_o^2} = \frac{2}{r_o^2}\int_0^{r_o} u(r,x) r dr \qquad 5.50$$



The velocity distribution for the fully developed flow in tube may be written

$\frac{u}{u_o} = 1 - \frac{r^2}{r_o^2}$ 5.51

The temperature distribution in the tube can calculated from the equation below:

$T - T_c = \frac{1}{\alpha}\frac{\partial T}{\partial x}\frac{u_o r_o^2}{4}\left[\left(\frac{r}{r_o}\right)^2 - \frac{1}{4}\left(\frac{r}{r_o}\right)^4\right]$ 5.52

Where:

T_c : center temperature.

The Bulk Temperature

In tube flow the convection heat-transfer coefficient is usually defined by

$\frac{q}{A} = h(T_w - T_b)$ (local heat flux) 5.53

Where

T_w :is the wall temperature and

T_b :is the so-called *bulk temperature,* or energy-average fluid temperature across the tube, which may be calculated from

$T_b = \bar{T} = \frac{\int_0^{r_0} \rho 2\pi r dr u c_p T}{\int_0^{r_0} \rho 2\pi r dr u c_p}$ 5.54

$T_b = T_c + \frac{7}{96}\frac{u_o r_o^2}{\alpha}\frac{\partial T}{\partial x}$ 5.55

and for the wall temperature

$T_w = T_c + \frac{3}{16}\frac{u_o r_o^2}{\alpha}\frac{\partial T}{\partial x}$ 5.56

The heat-transfer coefficient is calculated from

$q = hA(T_w - T_b) = kA\left(\frac{\partial T}{\partial r}\right)_{r=r_o}$ 5.57

$h = \frac{k\left(\frac{\partial T}{\partial r}\right)_{r=r_o}}{(T_w - T_b)}$ 5.58

The temperature gradient is given by

$$\left.\frac{\partial T}{\partial r}\right]_{r=r_o} = \frac{u_o}{\alpha}\frac{\partial T}{\partial x}\left(\frac{r}{2}-\frac{r^3}{4r_o^2}\right)_{r=r_o} = \frac{u_o r_o}{4\alpha}\frac{\partial T}{\partial x} \qquad 5.59$$

Substituting Equations (5.55), (5.56), and (5.59) in Equation (5.58) gives

$$h = \frac{24}{11}\frac{k}{r_o} = \frac{48}{11}\frac{k}{D} \qquad 5.60$$

Expressed in terms of the Nusselt number, the result is

$$Nu_D = \frac{hd}{k} = 4.364 \qquad 5.61$$

$$h = 4.364 * \frac{k}{d}$$

5.8.2 Turbulent Flow in a Tube

The developed velocity profile for turbulent flow in a tube will appear as shown in Figure 5.8.

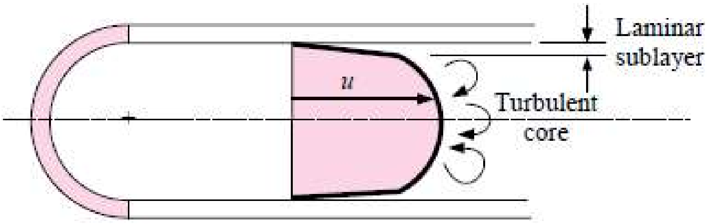


Figure 5.8 Velocity profile in turbulent tube flow.

$$St = \frac{h}{\rho c_p u_m} = \frac{Nu_D}{Re_D Pr} = \frac{f}{8} \qquad Pr \approx 1 \qquad 5.62$$

$$f = \frac{0.316}{Re_D^{\frac{1}{4}}} \qquad \text{for } 4*10^3 < Re_D \leq 2*10^5 \quad \text{and} \qquad 5.63$$

$$\frac{Nu_D}{Re_D Pr} = 0.0395 Re_D^{-\frac{1}{4}}$$

$$Nu_D = 0.0395 Re_D^{\frac{3}{4}} \qquad \text{for } 4*10^3 < Re_D \leq 2*10^5 \quad \text{and} \quad Pr \approx 1 \qquad 5.64$$

$$St Pr^{\frac{2}{3}} = \frac{f}{8} \qquad Pr \neq 1 \qquad 5.65$$

$$Nu_D = 0.0395 Re_D^{\frac{3}{4}} Pr^{\frac{1}{3}} \qquad \text{for } 4*10^3 < Re_D \leq 2*10^5 \text{ and } Pr \neq 1 \qquad 5.66$$

5.8.3 Empirical Relation for Flow In Tube